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DRDC Toronto Technical Note (TN) 2003-035

CONVERSION BETWEEN CENTRIFUGE SPEED & Gz

Introduction

The DRDC human centrifuge simulates acceleration of a high performance aircraft. The gondola is supported on the end of the rotating arm, and with a centre of gravity offset from the support axis, rolls freely and passively on a horizontal axis perpendicular to the arm centerline (see first schematic). At constant arm or motor speed, the total acceleration is the vector sum of centripetal and gravitational accelerations, and the line between the roll axis and the gondola centre of gravity aligns with this total acceleration direction. With a subject sitting in the gondola, this is the foot-to-head direction, defined as +Z, and the acceleration vector is called G_z.

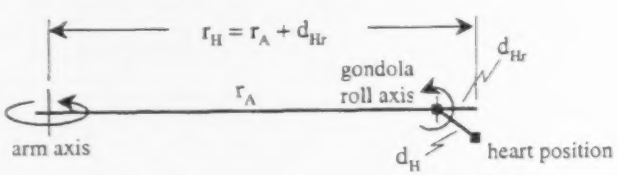
It is desired to control and measure G_z at the heart position, which is located about 1 ft below the roll axis. G_z is a function of arm speed and radius of rotation at the heart, in turn dependant on arm speed. The purpose of this note is to identify algorithms that can be used to convert between G_z and arm speed for use in centrifuge command and control. This includes assessment of accuracy.

Terminology

d_H	distance from heart to roll axis	r_H	radius of centrifuge rotation at heart
d_{Hr}	radial component of d_H	r_A	arm radius to gondola roll axis
G_g	acceleration of gravity	R	gear ratio
G_c	centripetal (or radial) acceleration	θ_G	gondola roll angle
G_z	headward acceleration at heart	ω_A	arm angular velocity
k	unit conversion factor	ω_M	motor angular velocity

Schematics

Arm & Gondola



θ_G vs d_H and G_z

DRDC TN 2003-035: Conversion between centrifuge speed & G_z

Given Values

$d_H = 1.0$ ft $G_g = 1$ g $= 32.17$ ft/s2 $k = \frac{2\pi}{60}\frac{rad/s}{RPM}$ ($\equiv 1$) $r_A = 20$ ft $R = 24.794$

Equations

$G_z = \sqrt{G_c^2 + G_g^2}$ where $G_c = \frac{r_H(k\omega_A)^2}{g}$ with ω_A in RPM

or $G_z = \sqrt{\frac{r_H^2(k\omega_A)^4}{g^2} + 1}$ 1

$r_H = r_A + d_{Hr}$ 2

where $d_{Hr} = d_H \sin\theta_G$ 3

and $\theta_G = \cos^{-1}\left(\frac{1}{G_z}\right)$ 4

$\omega_A = \frac{\omega_M}{R}$ 5

Numerical Determination of Rotation Radius at Heart

The value of r_H for each of several ω_A is found iteratively, as follows:

- Any value of ω_A is chosen, and $d_{Hr} = 0$ is assumed
- r_H is then estimated from equation 2, G_z from equation 1, θ_G from equation 4 and a new d_{Hr} from equation 3
- This iteration is repeated n times, until estimated values stabilize

Example: Let $\omega_A = 11.988$ RPM (corresponds to $G_z = 1.400$ g for $r_H = 20$ ft)

Values stablize after just a few iterations (5 shown here).

n	d_{Hr}	r_H	G_z	θ_G	(new) d_{Hr}
0	0	20	1.400	44.4	0.6998
1	0.6998	20.6998	1.424	45.4	0.7120
2	0.7120	20.7120	1.425	45.4	0.7122
3	0.7122	20.7122	1.425	45.4	0.7122
4	0.7122	20.7122	1.425	45.4	0.7122

Results — Arm Speed vs Heart *Position*, Acceleration and Roll (from Annex A)

ω_A (RPM)	r_H (ft)	G_z (g)	θ_G (deg)
5.048	20.173	1.015	9.9
8.198	20.424	1.104	25.1
9.864	20.564	1.211	34.3
11.988	20.712	1.425	45.4
15.940	20.875	2.066	61.1
20.369	20.947	3.127	71.3
25.369	20.977	4.709	77.7
29.459	20.987	6.288	80.8
33.020	20.992	7.866	82.7
36.222	20.994	9.442	83.9
38.204	20.995	10.493	84.5
42.752	20.997	13.119	85.6
46.856	20.998	15.745	86.4

Algorithms

A polynomial fit of arm speed and acceleration results above gives the following formulas. Note equation 5 relates motor and arm speeds.

$$G_z = c_1\omega_A^2 + c_2\omega_A + c_3 \quad \text{..... 6}$$

and $$\omega_A = \frac{c_2 + \sqrt{c_2^2 - 4c_1(c_3 - G_z)}}{c_1} \quad \text{..... 7}$$

where $c_1 = 0.0079095$

$c_2 = -0.054593$

$c_3 = 1.00000$

Sample conversion tables from equations 6 and 7 are included as Annex B.

Accuracy of Calculated Acceleration

The table and graph below compare original G_z data and equation 6 for each arm speed. Maximum curve fit error alone, approaching 0.1 g, occurs only at a G_z value of just under 1.1 g, outside the training and research operating envelope of 1.4 (Base) to 15 g. The maximum of the other three local peak errors is well below (about $^1/_3$) the steady G_z target tracking tolerance, which is ±0.2 g (Centrifuge Requirements Document).

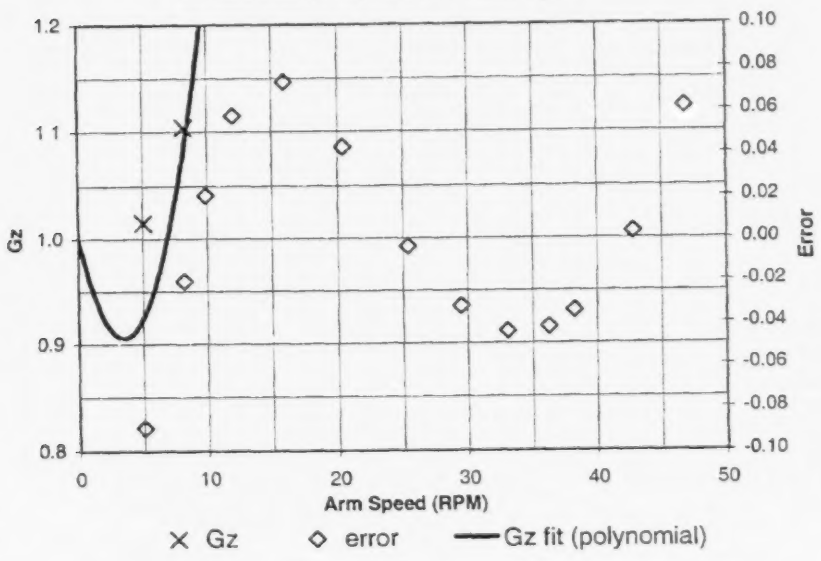
With G_z proportional to r_H (max. 252 inches), any discrepancy between given d_H (1ft) and actual adds another G_z error of ~0.4% (e.g., 0.04 g at 10 g) per inch in the r_A/r_H direction.

Gz — Data vs Calculated

	---------- G_z Value ----------		
ω_A	Data	Calculated (from 6)	Error *
5.048	1.015	0.926	**-0.089**
8.198	1.104	1.084	-0.020
9.864	1.211	1.231	0.021
11.988	1.425	1.482	0.058
15.940	2.066	2.139	**0.073**
20.369	3.127	3.170	0.043
25.369	4.709	4.705	-0.004
29.459	6.288	6.255	-0.032
33.020	7.866	7.822	**-0.044**
36.222	9.442	9.400	-0.042
38.204	10.493	10.458	-0.034
42.752	13.119	13.122	0.003
46.856	15.745	15.807	**0.062**

• Values in **bold** are approximate local peak errors (see graph also)

Gz and Curve-fit Error vs Arm Speed



Conclusion and Recommendation

Equations 6 and 7 enable conversion between G_z and centrifuge arm speed. With conversion error well below steady G_z tracking tolerance, these formulas are considered acceptable and recommended for use in command and control of the centrifuge.

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Annexes

A. Numerical calculation for rotation radius at heart for selected arm speeds

B. Sample conversion tables

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ANNEX A to TN 2003-035:

Numerical Calculation for Rotation Radius at Heart at Selected Arm Speeds

n	d_{Hr} (ft)	r_H (ft) (Eqn 2)	G_z (g) (Eqn 1)	θ_G (deg) (Eqn 4)	new d_{Hr} (ft) (Eqn 3)
RPM =	**5.048**				
0	0	20.0000	1.015	9.85	0.1712
1	0.1712	20.1712	1.015	9.94	0.1726
2	0.1726	20.1726	1.015	9.94	0.1726
3	0.1726	20.1726	1.015	9 94	0.1726
4	0.1726	20.1726	1.015	9.94	0.1726
RPM =	**8.198**				
0	0	20.0000	1.100	24.62	0.4165
1	0.4165	20.4165	1.104	25.07	0.4237
2	0.4237	20.4237	1.104	25.07	0.4238
3	0.4238	20.4238	1.104	25.07	0.4238
4	0.4238	20.4238	1 104	25.07	0.4238
RPM =	**9.864**				
0	0	20.0000	1.200	33.56	0.5528
1	0.5528	20.5528	1.210	34.28	0.5632
2	0.5632	20.5632	1 210	34.29	0.5634
3	0.5634	20.5634	1.210	34.29	0.5634
4	0.5634	20.5634	1.210	34 29	0 5634
RPM =	**11.988**				
0	0	20.0000	1.400	44.41	0.6998
1	0.6998	20.6998	1 424	45.40	0.7120
2	0.7120	20.7120	1.425	45.42	0.7122
3	0.7122	20.7122	1.425	45 42	0.7122
4	0.7122	20.7122	1 425	45.42	0.7122
RPM =	**15.940**				
0	0	20.0000	2.000	60 00	0.8660
1	0.8660	20.8660	2.065	61.04	0 8750
2	0.8750	20.8750	2.066	61.05	0.8750
3	0.8750	20.8750	2.066	61.05	0.8750
4	0.8750	20.8750	2.066	61.05	0.8750
RPM =	**20.369**				
0	0	20.0000	3.000	70.53	0 9428
1	0.9428	20.9428	3.126	71.34	0.9475
2	0.9475	20.9475	3.127	71.35	0.9475
3	0.9475	20.9475	3.127	71.35	0.9475
4	0.9475	20.9475	3.127	71.35	0.9475
RPM =	**25.369**				
0	0	20.0000	4 500	77.16	0.9750
1	0.9750	20.9750	4.709	77.74	0.9772
2	0.9772	20.9772	4.709	77.74	0.9772
3	0.9772	20.9772	4.709	77.74	0.9772
4	0.9772	20.9772	4.709	77 74	0 9772

ANNEX A to TN 2003-035:
Numerical Calculation for Rotation Radius at Heart at Selected Arm Speeds

n	d_{Hr} (ft)	r_H (ft) (Eqn 2)	G_z (g) (Eqn 1)	θ_G (deg) (Eqn 4)	new d_{Hr} (ft) (Eqn 3)
RPM =	**29.459**				
0	0	20 0000	6.000	80.41	0.9860
1	0.9860	20.9860	6.288	80.85	0.9873
2	0.9873	20.9873	6.288	80.85	0.9873
3	0.9873	20.9873	6.288	80.85	0.9873
4	0.9873	20.9873	6.288	80.85	0.9873
RPM =	**33.020**				
0	0	20.0000	7.500	82.34	0.9911
1	0.9911	20.9911	7.865	82.70	0.9919
2	0.9919	20 9919	7.865	82 70	0.9919
3	0.9919	20.9919	7.865	82 70	0.9919
4	0.9919	20.9919	7 865	82.70	0.9919
RPM =	**36.222**				
0	0	20.0000	9.000	83 62	0.9938
1	0.9938	20.9938	9.442	83 92	0.9944
2	0.9944	20.9944	9.442	83.92	0.9944
3	0.9944	20.9944	9.442	83.92	0.9944
4	0.9944	20.9944	9.442	83.92	0 9944
RPM =	**38.204**				
0	0	20.0000	10.000	84.26	0.9950
1	0.9950	20.9950	10.493	84.53	0.9954
2	0.9954	20.9954	10.493	84 53	0.9954
3	0.9954	20.9954	10.493	84.53	0.9954
4	0.9954	20 9954	10 493	84 53	0.9954
RPM =	**42.752**				
0	0	20.0000	12.500	85.41	0 9968
1	0.9968	20.9968	13.119	85 63	0.9971
2	0.9971	20.9971	13.119	85.63	0.9971
3	0.9971	20.9971	13.119	85.63	0.9971
4	0.9971	20.9971	13.119	85.63	0.9971
RPM =	**46.856**				
0	0	20.0000	15.000	86 18	0.9978
1	0.9978	20.9978	15.745	86.36	0.9980
2	0.9980	20.9980	15.745	86.36	0.9980
3	0.9980	20.9980	15.745	86.36	0.9980
4	0 9980	20.9980	15 74522	86.36	0.9980

ANNEX B to TN 2003-035:
Sample Conversion Tables

Equation 6

$$G_z = c_1 RPM^2 + c_2 RPM + c_3$$

c1 = 0.0079095 c2 = -0.054593 c3 = 1.00000

arm RPM	Gz
4	0.91
6	0.96
8	1.07
10	1.25
12	1.48
14	1.79
16	2.15
18	2.58
20	3.07
22	3.63
24	4.25
26	4 93
28	5.67
30	6.48
32	7 35
34	8.29
36	9.29
38	10.35
40	11.47
42	12.66
44	13.91
46	15.23
48	16.60

Equation 7

$$RPM = \frac{c_2 \pm \sqrt{c_2^2 - 4c_1(c_3 - G_z)}}{c_1}$$

Gz	arm RPM
1.20	9.55
1 30	10.51
1.40	11.36
1.50	12.12
1.60	12.82
1.80	14.08
2.00	15.21
2.25	16.49
2.50	17.65
2.75	18.72
3.00	19.72
3.25	20.67
3.50	21.56
3.75	22.41
4.00	23.23
4.25	24.01
4.50	24.77
4.75	25.50
5.00	26.20
5.25	26.89
5 50	27.55
5.75	28.20
6.00	28.83
6.50	30.05
7.00	31.21
7.50	32.33
8.00	33.40
8.50	34.44
9.00	35.44
9.50	36.41
10.00	37.36
10.50	38.28
11.00	39.18
11.50	40.05
12.00	40.90
13.00	42.55
14.00	44.14
15.00	45.66
16.00	47.14

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